



Basic Garden Math

Gardening is an activity which occasionally requires the use of math, such as when you are computing how much fertilizer to use or how much compost to buy. Luckily, the math involved is simple and easily applied.

All that is probably necessary for most readers is a review of the basic principles and a step-by-step description of their application. The best advice is to start slowly and carefully, draw simple diagrams to help you keep track of things, and use a calculator to avoid arithmetic mistakes. It will become much easier after a little practice.

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Squares

Squares are four-sided shapes where all sides are the same length and all angles are 90° .

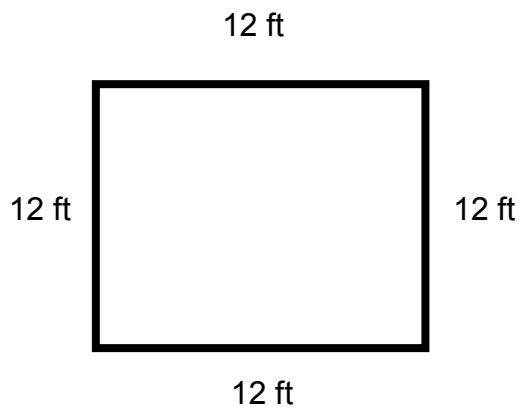
Some Basic Definitions:

Area: The two-dimensional space covered by a shape or object. It is usually expressed as the square of some linear measurement, such as square foot (ft^2) or square meter (m^2). Terms such as acre and hectare are also used to describe areas.

Perimeter: The one-dimensional or linear distance along the boundary of a shape or object. Gardeners often use this to determine the length of garden borders.

Formula 1: **Square Area = Side²**

Formula 2: **Square Perimeter = Side x 4**



Square Example

Side = 12 ft

Square Area (Formula 1) = Side²
Area = 12 ft x 12 ft
Area = 144 ft²

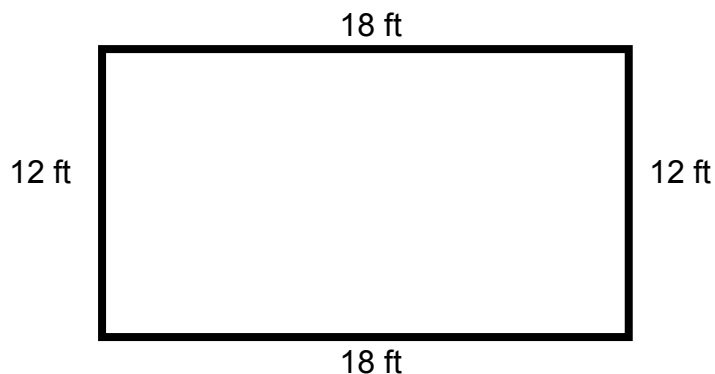
Perimeter (Formula 2) = Side x 4
Perimeter = 12 ft x 4
Perimeter = 48 ft

Rectangles

Rectangles are four-sided shapes, with two opposite sides of one length, and the remaining two opposite sides of another length. All of the angles are right angles (90°).

Formula 3: **Rectangle Area = Longer Side x Shorter Side**

Formula 4: **Rectangle Perimeter = (Longer Side x 2) + (Shorter Side x 2)**



Rectangle Example

Longer Sides = 18 ft

Shorter Sides = 12 ft

Rectangle Area (Formula 3) = Longer Side x Shorter Side

Area = 18 ft x 12 ft

Area = 216 ft²

Rectangle Perimeter (Formula 4) = (Longer Side x 2) + (Shorter Side x 2)

Perimeter = (18 ft x 2) + (12 ft x 2)

Perimeter = 36 ft + 24 ft

Perimeter = 60 ft

Triangles

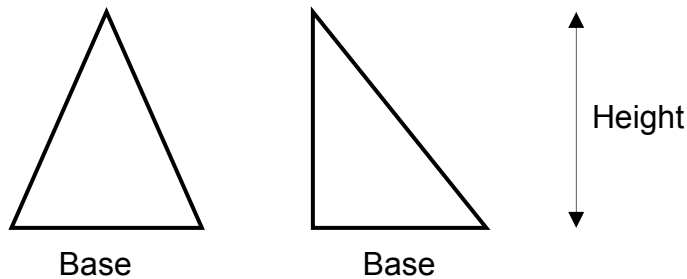
Triangles are shapes with three sides.

Triangle definitions:

Right Triangle: A triangle that has one angle equal to exactly 90° .

Base: Any one of the triangle sides (it is often rotated for convenience so that it is horizontal across the page, but this is not necessary). The base is used to compute the area of the triangle.

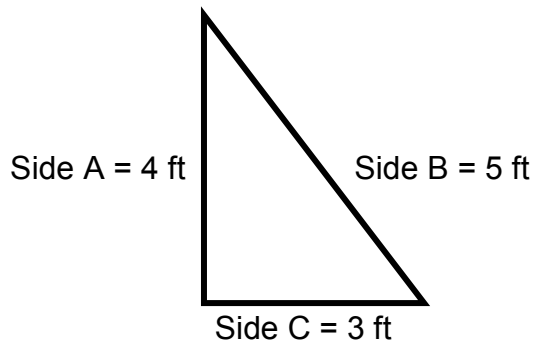
Height: The perpendicular distance from the base to the angle created by the other two sides. The height is used to compute the area of the triangle.



(Note: While they have significantly different shapes, the two triangles above have equal bases and heights, and from Formula 5 below we see that they have the same area. However, they do not have the same perimeter. The triangle on the right is a right triangle.)

Formula 5: Triangle Area = Base x Height / 2

Formula 6: Triangle Perimeter = Side A + Side B + Side C



Triangle Example

Side A = 4 ft

Side B = 5 ft

Side C = 3 ft

Base = 3 ft (Side C in this example)

Height = 4 ft (Side A in this example)

Triangle Area (Formula 5) = Base x Height / 2

Area = 3 ft x 4 ft / 2

Area = 12 ft² / 2

Area = 6 ft²

Triangle Perimeter (Formula 6) = Side A + Side B + Side C

Perimeter = 4 ft + 5 ft + 3 ft

Perimeter = 12 ft

(Note: A triangle whose sides have a ratio of 3:4:5, like the one in the example above, is very useful. If you lay out such a triangle with stakes and string in your garden, the angle formed between the sides with lengths of 3 and 4 is a right angle of exactly 90°. The units of measure are not important: a 3:4:5 triangle measured off in yards has the same proportional shape, and the same perfect 90° angle, as a 3:4:5 triangle measured off in either inches or meters.)

Circles

Circles are round shapes where every point on the circle is the same distance from a single point in the center.

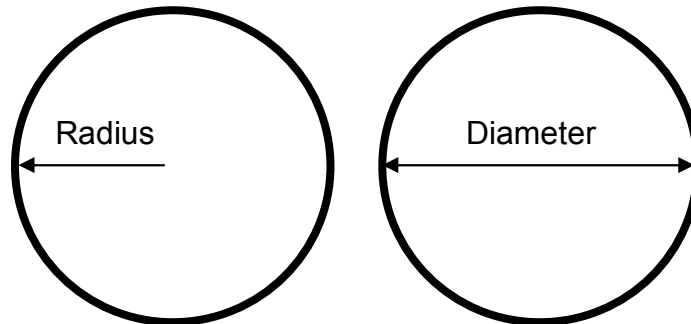
Circle definitions:

Circumference: The distance around a circle. This is the same as the perimeter of the circle.

Diameter: The straight-line distance across a circle at its widest point. One half of the diameter of a circle is the radius of that circle. The diameter always passes through the center of a circle.

Radius: The straight-line distance from the center of a circle to the edge of the circle. Twice the radius of a circle is the diameter of that circle.

pi (π): pi is the ratio between the diameter of a circle and the circumference of that circle. In other words, if you take the circumference of a circle and divide it by the diameter of that circle, you get pi. pi is the same for all circles regardless of their size, and for our purposes you can consider it to always equal 3.1416.

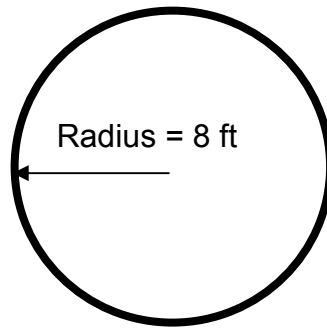


Formula 7a: **Circle Area = $\pi \times \text{Radius}^2$**

Formula 7b: **Circle Area = $\pi \times \text{Diameter}^2 / 4$**

Formula 8a: **Circle Circumference = $\pi \times \text{Radius} \times 2$**

Formula 8b: **Circle Circumference = $\pi \times \text{Diameter}$**



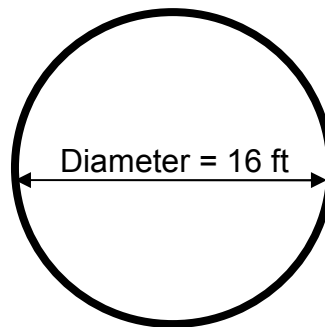
Circle Example 1

$$\pi = 3.1416$$

$$\text{Radius} = 8 \text{ ft}$$

$$\begin{aligned}\text{Circle Area (Formula 7a)} &= \pi \times \text{Radius}^2 \\ \text{Area} &= 3.1416 \times (8 \text{ ft} \times 8 \text{ ft}) \\ \text{Area} &= 3.1416 \times 64 \text{ ft}^2 \\ \text{Area} &= 201.06 \text{ ft}^2\end{aligned}$$

$$\begin{aligned}\text{Circle Perimeter (Formula 8a)} &= \pi \times \text{Radius} \times 2 \\ \text{Perimeter} &= 3.1416 \times 8 \text{ ft} \times 2 \\ \text{Perimeter} &= 50.27 \text{ ft}\end{aligned}$$



Circle Example 2

$$\pi = 3.1416$$

$$\text{Diameter} = 16 \text{ ft}$$

$$\begin{aligned}\text{Circle Area (Formula 7b)} &= \pi \times \text{Diameter}^2 / 4 \\ \text{Area} &= 3.1416 \times (16 \text{ ft} \times 16 \text{ ft}) / 4 \\ \text{Area} &= 3.1416 \times 256 \text{ ft}^2 / 4 \\ \text{Area} &= 201.06 \text{ ft}^2\end{aligned}$$

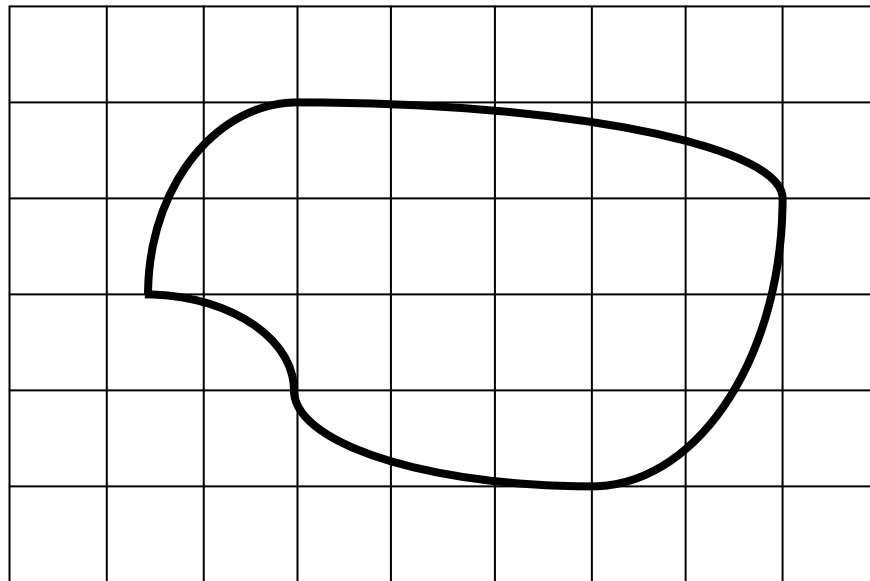
$$\begin{aligned}\text{Circle Perimeter (Formula 8b)} &= \pi \times \text{Diameter} \\ \text{Perimeter} &= 3.1416 \times 16 \text{ ft} \\ \text{Perimeter} &= 50.27 \text{ ft}\end{aligned}$$

Irregular Shapes

Life would be easy if all we had to deal with in the garden were simple geometric shapes like squares, rectangles, triangles, and circles. When dealing with irregular shapes in the garden, there are two relatively simple (but not the only) approaches: 1) using a grid overlay to estimate the area, and 2) calculating the area and perimeter by “exploding” an irregular shape into simpler geometric shapes.

Using A Grid Overlay.

This method involves placing a grid of known scale over a drawing of the irregular shape. It doesn't really matter what grid scale is used as long as it is uniform. This method will give you an estimate of the area of an irregular shape, but will not give you the perimeter measurement.



Scale: Grid Cells = 1 ft²

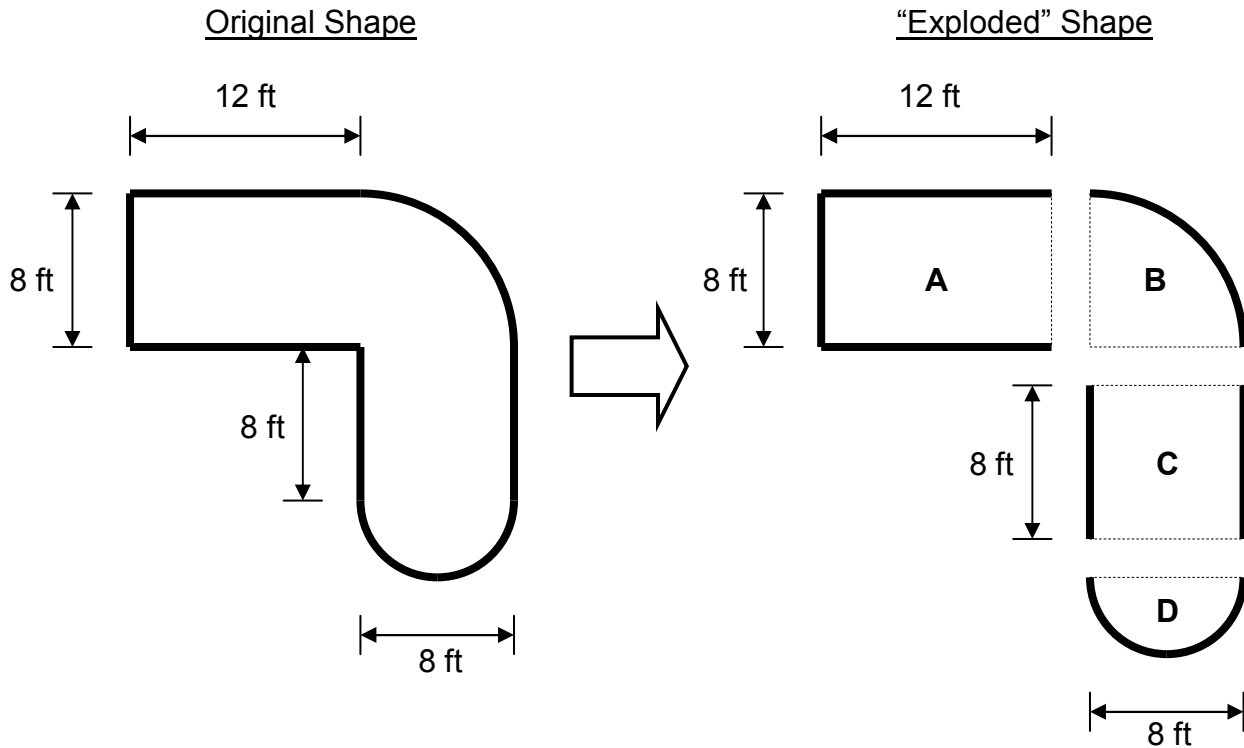
To estimate the area of the irregular shape above, look at each one of the grid cells and see how much of it is occupied (or covered) by the shape. You will assign a value to each of the grid cells: if all of a cell is occupied by the shape it has a value of 1, if half of the cell is occupied the cell has a value of $\frac{1}{2}$ or 0.5, if only one quarter of the cell is occupied it has a value of $\frac{1}{4}$ or 0.25, and so on. Cells that are not occupied by any part of the shape have a value of 0 and can be ignored.

When you have established a value for each of the cells occupied by the shape, add them all together to get an estimate of the irregular shape's area (don't forget to attach the scale units). In the irregular shape above the estimated area is approximately 19.75 ft². Do you get the same area estimate?

To determine the perimeter of this shape, one method would be to run a string along the shape outline, and then remove the string and measure its length.

“Exploding” An Irregular Shape Into Simpler Shapes.

This method involves breaking up a single irregular shape into two or more simpler geometric shapes. The following example is relatively complex:



Shape	Initial Area Calculations	Adjustments	Result
Rectangle A	Use Formula 3: Area = 8 ft x 12 ft = 96 ft ²	None.	96.0 ft ²
Quarter-circle B	Use Formula 7a: Area = 3.1416 x (8 ft) ² = 201.1 ft ²	Only ¼ of the calculated area is part of the total area.	50.3 ft ²
Square C	Use Formula 1: Area = 8 ft x 8 ft = 64 ft ²	None.	64.0 ft ²
Semi-circle D	Use Formula 7b: Area = 3.1416 x (8 ft) ² / 4 = 50.3 ft ²	Only ½ of the calculated area is part of the total area.	25.1 ft ²

Total Area of the Original Shape: 235.4 ft²

Shape	Initial Perimeter Calculations	Adjustments	Result
Rectangle A	Use Formula 4: Perimeter = (12 ft x 2) + (8 ft X 2) = 40 ft	One of the short (8 ft) sides of the rectangle is not part of the perimeter.	32.0 ft
Quarter-circle B	Use Formula 8a: Circumference = 3.1416 x 8 ft x 2 = 50.3 ft	Only ¼ of the calculated circumference is part of the perimeter.	12.6 ft
Square C	Use Formula 2: Perimeter = 8 ft x 4 = 32 ft	Only two sides of the square are included in the perimeter.	16.0 ft
Semi-circle D	Use Formula 8b: Circumference = 3.1416 x 8 ft = 25.1 ft	Only ½ of the calculated circumference is part of the perimeter.	12.6 ft

Total Perimeter of the Original Shape: 73.2 ft

Volumes

We have seen how to compute one-dimensional or linear garden distances, such as the perimeter of geometric shapes. We have also worked with areas, the two-dimensional shapes that have both width and length. Volumes are three-dimensional objects, where one considers not only the physical area covered, but also the height or thickness of the coverage.

A volume is the space contained within a three-dimensional object or space. It is usually expressed as the cube of some linear measurement, such as cubic yard (yd^3) or cubic centimeter (cc or cm^3), but terms such as gallon and liter are also used to describe volumes.

The gardener most often deals with volumes when he or she is adding mulch, compost, or a bulk soil amendment to a bed. The instructions will often say to cover the soil with 3 inches or 6 inches of some material. Bulk materials vary greatly in weight, so most are sold by volume, such as the cubic foot (ft^3) or cubic yard (yd^3). In order to purchase the correct amount of bulk material, you need to determine what volume of material you need.

Follow these steps to determine the volume of material needed to cover an area to a specified depth:

1. Calculate the area of the space you wish to cover. Use the formulas and examples presented above to calculate or estimate the area to be covered. For simplicity, whenever possible do your measurements and calculations in the units used to supply the bulk material. In other words, if the bulk material is sold by the cubic foot, do your calculations in feet rather than yards so there is no need to convert yards to feet later.
2. After you have calculated the area to be covered, multiply the area by the depth desired (using the same units of measure). This is where you turn a two-dimensional shape (area) into a three-dimensional volume.

Warning: Make sure you are using the same basic units of measure for both the area and the depth of the material! If you measured the area in ft^2 , then you must use feet (or a fraction of a foot) when you do this step or your answer will be incorrect.

For example, if you want to cover a 100 ft^2 area to a depth of 3 inches, multiply the 100 ft^2 area by 0.25 ft (because 3 inches is 25% of a foot), giving you the correct answer that 25 ft^3 of material will be needed. If you were to multiply the 100 ft^2 directly by 3, you would get the incorrect answer of 300 ft^3 of material, which is 12 times more material than you need!

3. Consider how the material you wish to purchase is supplied. Using the 25 ft^3 example above:

Volume Needed	How Supplied	Calculations	What To Purchase
25 ft^3	Bulk Delivery in yd^3 .	$1 \text{ yd}^3 = 27 \text{ ft}^3$	Pick up or order delivery of 1 yd^3 (2 ft^3 will be left over).
25 ft^3	$2/3 \text{ yd}^3$ bags.	$1 \text{ yd}^3 = 27 \text{ ft}^3$ $2/3 \text{ yd}^3 = 18 \text{ ft}^3$ $25 \text{ ft}^3 / 18 \text{ ft}^3 = 1.4 \text{ bags}$	Purchase 2 of the $2/3 \text{ yd}^3$ bags (11 ft^3 will be left over).

Converting Measurements

There are instances when you will need to convert from one measurement unit to another. Examples are when you need the number of square feet in a square yard, or to convert pounds to kilograms. The table below gives some useful equivalents for gardeners.

Some examples:

- To convert 13.5 L to gal: multiply 13.5 L x 0.264 to get 3.56 gal.
- To convert 6 m² to yd²: multiply 6 m² by 1.196 to get 7.18 yd².
- To convert 100 lbs to Kg: first multiply 100 lbs by 453.6 to get 45360 g, then divide 45360 g by 1000 to get the final answer of 45.36 Kg.

Weight					
1 ounce (oz) avdp.	=	28.35 g	1 gram (g)	=	0.0353 oz avdp.
1 pound (lb)	=	453.6 g	1 kilogram (kg)	=	2.205 lb
1 short ton	=	0.9078 metric ton	1 metric ton	=	1.1016 short tons
Volume					
1 cubic inch (in ³)	=	16.39 ml	1 milliliter (ml)	=	0.0610 in ³
1 cubic foot (ft ³)	=	28.32 L	1 liter (L)	=	61.02 in ³
		7.48 gal (US)			264.2 gal (US)
1 cubic yard (yd ³)	=	27 ft ³	1 cubic meter (m ³)	=	35.3 ft ³
		0.765 m ³			1.308 yd ³
Liquid Measure					
1 teaspoon (tsp)	=	5 ml	1 ml	=	0.034 fl oz
1 Tablespoon (Tbsp)	=	3 teaspoons (tsp)	1 L	=	1.057 qt (US)
1 fluid ounce (fl oz)	=	29.6 ml			0.264 gal (US)
1 quart (qt) (US)	=	946.3 ml			
1 gallon (gal) (US)	=	3.785 L			
Area					
1 square inch (in ²)	=	6.452 cm ²	1 square cm (cm ²)	=	0.155 in ²
1 square foot (ft ²)	=	929.09 cm ²	1 sq meter (m ²)	=	10.76 ft ²
		43,560 ft ²			1.196 yd ²
1 acre	=	0.4047 ha	1 hectare (ha)	=	2.471 acres
					1,000 m ²
Length					
1 inch (in)	=	2.54 cm	1 cm	=	0.394 in
1 foot (ft)	=	30.48 cm	1 m	=	39.37 in
1 yard (yd)	=	91.44 cm			3.281 ft
		0.9144 m			1.094 yd
Temperature					
°F	=	(°C x 9/5) + 32	°C	=	(°F - 32) x 5/9

Typical Garden Situations

Note: Fertilizer and soil amendment application instructions are usually given as a rate: a specific amount of product per specified area. An example would be an application rate of 3 lbs of product per 100 ft² of area to be covered.

Example #1.

The soil pH of your established blueberry patch is too high. You have been advised by the soil testing laboratory to lower the soil pH by applying a topdressing of elemental sulfur at a rate of 10 lbs of sulfur per 1000 ft². Your blueberry patch is 25 ft wide by 50 ft long. How much sulfur do you need to purchase and use?

- Determine the size of the area to be treated. You want to treat a 25 ft x 50 ft rectangle, so use Formula 3: $\text{area} = 25 \text{ ft} \times 50 \text{ ft} = 1250 \text{ ft}^2$.
- Compare your area with the application rate area. The recommended amount of sulfur to be used is based on a 1000 ft² area. $1250 \text{ ft}^2 / 1000 \text{ ft}^2 = 1.25$. Your area is 1.25 times larger.
- Determine the amount of product to apply. Since your area is 1.25 times larger than the application rate area, you will need 1.25 times the specified amount of sulfur:
 $10 \text{ lbs sulfur} \times 1.25 = 12.5 \text{ lbs sulfur}$.

Answer: You will need to purchase 12.5 lbs of sulfur to cover your 1250 ft² blueberry patch at a rate of 10 lbs of sulfur per 1000 ft² of soil.

Example #2.

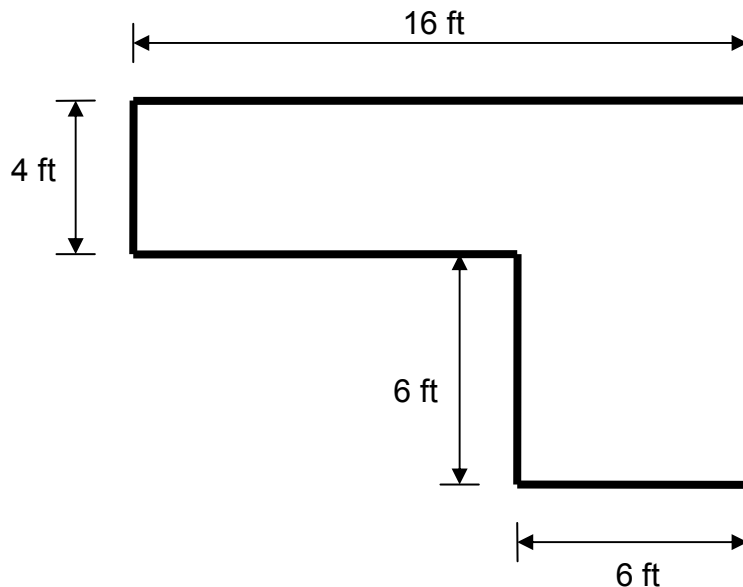
The soil testing laboratory has indicated you need to raise the soil pH around your fruit tree with an application of 7 cups of ground limestone per 100 ft² of soil being treated. You want to amend the soil in a 15 ft circle out from your tree. How much ground limestone do you need?

- Determine the size of the area to be treated. In this example the tree trunk represents the center point of a circle with a 15 ft radius. Use Formula 7a to find the area of this circle: $\text{area} = 3.1416 \times (15 \text{ ft})^2 = 707 \text{ ft}^2$.
- Compare your area with the application rate area. The recommended amount of limestone to be used is based on a 100 ft² area. $707 \text{ ft}^2 / 100 \text{ ft}^2 = 7.07$. Your area is 7.07 times larger.
- Determine the amount of product to apply. Since your area is 7.07 times larger than the application rate area, you will need 7.07 times the specified amount of limestone:
 $7 \text{ cups limestone} \times 7.07 = 49.5 \text{ cups limestone}$.

Answer: You will need to purchase 49.5 cups of limestone to cover a 15 ft radius circle around your tree at a rate of 7 cups of limestone per 100 ft² of soil.

Example #3.

You want to cover the following area with 4 inches of compost, which is supplied in $\frac{2}{3}$ yd³ bags:



- Determine the total area. In this example you could break the irregular area into two simpler shapes: a rectangle which is 4 ft by 16 ft, and a square that is 6 ft by 6 ft. The area of the rectangle can be found using Formula 3: $\text{area} = 4 \text{ ft} \times 16 \text{ ft} = 64 \text{ ft}^2$. The area of the square can be found using Formula 1: $\text{area} = (6 \text{ ft})^2 = 36 \text{ ft}^2$. Add these two areas together to get the total area: $64 \text{ ft}^2 + 36 \text{ ft}^2 = 100 \text{ ft}^2$.
- Determine the volume of material needed. Since the area measurements were done in feet, you should use feet when calculating the volume. 4 inches of compost is equal to 0.33 ft of compost (4 inches is $\frac{1}{3}$ of a foot). Multiply the total area by the desired depth to get the volume of compost needed: $100 \text{ ft}^2 \times 0.33 \text{ ft} = 33 \text{ ft}^3$.
- Determine the amount of product to purchase. Since $1 \text{ yd}^3 = 27 \text{ ft}^3$, each $\frac{2}{3}$ yd³ bag of compost contains 18 ft^3 of compost ($\frac{2}{3}$ of 27 is 18). Dividing the volume you need by how it is supplied will give you the number of $\frac{2}{3}$ yd³ bags needed: $33 \text{ ft}^3 / 18 \text{ ft}^3 = 1.83$ bags.

Answer: Since partial bags are not sold, you will need to purchase two $\frac{2}{3}$ yd³ bags of compost to cover a 100 ft^2 area to a depth of 4 inches (with 3 ft^3 of the compost left over).

Note: On packages of commercial fertilizer the percentage by weight of nitrogen (N), phosphorous (P), and potassium (K) is shown with the N-P-K label. For example, a 10-6-4 fertilizer contains 10% nitrogen, 6% phosphorous, and 4% potassium by weight, whereas a fertilizer with a label of 42-0-0 (such as urea) contains 42% nitrogen by weight but no phosphorous or potassium. The fact that the N-P-K label lists percentages by weight is important, for if you have a 100 lb bag of a fertilizer you can read directly how many lbs of these nutrients it contains. For example, a 100 lb bag of 42-0-0 fertilizer contains 42 lbs of nitrogen ($100 \text{ lbs} \times 0.42 = 42 \text{ lbs}$).

Example #4.

You want to place 10-6-4 fertilizer on 300 ft^2 of your garden at a rate of 3 lb per 100 ft^2 . How much of this fertilizer will you need?

- Compare your area with the application rate area. Divide your area by the application rate area: $300 \text{ ft}^2 / 100 \text{ ft}^2 = 3$. Your area is 3 times larger.
- Determine how much of the 10-6-4 fertilizer to use. Since your area is 3 times larger than the application rate area, you will need 3 times the specified amount of fertilizer: $3 \text{ lb} \times 3 = 9 \text{ lbs}$.

Answer: You will need 9 lbs of 10-6-4 fertilizer to treat 300 ft^2 of garden at a rate of 3 lbs of fertilizer per 100 ft^2 .

Example #5.

You have a triangular plot, with a base of 10 ft and a height of 8 ft, and you wish to fertilize at a rate of 1 lb of nitrogen per 1000 ft^2 . The 100 lb bag of fertilizer you have on hand has a label of 12-0-0. How much of this fertilizer should you use on this area?

- Determine the size of the area to be treated. Use Formula 5 to find the area of this triangle: $\text{area} = 10 \text{ ft} \times 8 \text{ ft} / 2 = 40 \text{ ft}^2$.
- Compare your area with the application rate area. The amount of nitrogen desired is based on a 1000 ft^2 area. Divide your area by the application rate area: $40 \text{ ft}^2 / 1000 \text{ ft}^2 = 0.04$. Your area is 0.04 (or 4%) of the application rate area.
- Determine the amount of nitrogen to apply. Since you are going to fertilize only 4% of the application rate area, you only will need 4% of the 1 lb of nitrogen. Multiply the 1 lb of nitrogen by your percentage of the application rate area: $1 \text{ lb nitrogen} \times 0.04 = 0.04$ lbs of nitrogen is needed for your area.
- Determine how much of the 12-0-0 fertilizer to use. In the 100 lb bag of 12-0-0 fertilizer there is 12 lbs of nitrogen, or 0.12 lbs of nitrogen per lb of fertilizer. Divide the desired amount of nitrogen by the nitrogen in each lb of fertilizer: $0.04 \text{ lb} / 0.12 \text{ lb} = .33 \text{ lbs}$.

Answer: Use $1/3$ of a lb of the 12-0-0 fertilizer on the 40 ft^2 triangular area to fertilize at a rate of 1 lb of nitrogen per 1000 ft^2 .